As shown by curve f) in Fig. 1 the removal of the blade between the high and low energy channel [scheme f) in Fig. 2] ruins the diffuser performance. Noteworthy in this case is that the kinetic energy peak moves to the channel next to the combined channels. The promixity of the peak kinetic energy to the channel with the lowest energy seems to be an inherent feature of a bladed diffuser system as shown here.

The present diffuser tests show that for supersonic diffusers with high aspect ratio rectangular cross sections, energy redistribution by means of an internal blade system has a very beneficial effect on the diffusion process. Internal redistribution of kinetic energy might also be applicable in cases where a supersonic diffuser is subdivided into cells to shorten its length.[2] In this case diffuser cells located at the periphery may have to ingest the heavy boundary layer of an approach duct. These disadvantaged cells could possibly be energized by internal injection as shown in the present tests in an elemental way.

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Shock Waves in Charged Particle-Gas Mixtures

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Nomenclature

C_D = drag coefficient
C_{pg} = gas heat capacity
C_p = material heat capacity
E = electric field
H = convective heat transfer coefficient
k = gas thermal conductivity
K = particle to gas mass flow ratio
$\dot{m}$ = gas mass flow rate
M = particle mass
n_p = particle number density
N = Nusselt number
P = pressure
Q = charge per particle
R = perfect gas constant
Re = particle Reynolds number
T = gas temperature
T_p = particle temperature
u_p = particle velocity
x = distance, positive downstream
ε_o = dielectric constant of free space
ρ = gas density
ρ' = mass density of particulate material
ρ_p = particle mass density
σ = particle radius
ϕ = electric potential

Subscripts
$+\infty$ = downstream asymptotic state
$-\infty$ = upstream asymptotic state
i = particle species

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Introduction

When a dust particle traveling in a supersonic gas stream passes through a normal shock wave, aerodynamic drag causes the particle to decelerate to the gas velocity downstream of the shock as shown by Hoenig,[1] who examined the time and distance for a particle to reach a certain fraction of its initial excess velocity. An appreciable concentration of dust will affect gas motion behind the shock front, thus modifying the relaxation process. The effect of a particle suspension on the structure of a normal shock wave was first described by Carrier.[2] Subsequently, numerous extensions have appeared, some of which take into account mass transfer,[3,4] finite particle volume,[5,6] and radiation effects.[7]

A previous Note[8] reported experimental evidence for particle charging by collision in the relaxation zone behind the shock front. The purpose of the present work is to examine structure of the relaxation zone when the particles are initially charged. In a suspension, particles of different masses may acquire opposite charges so that the difference in aerodynamic drag produces a net charge separation. The electric field generated by this charge separation couples the motions of the dust particles, thereby altering the structure of the relaxation zone. In practice, charges could arise from several sources such as static charging by contact,[9] droplet breakup,[10] or they could be deliberately given an initial charge. Since a system of charged particles is obviously not stable, we are assuming that the shock transition takes place soon after the particles are charged. Thus the motion of the particles is strictly one-dimensional without collisions and neutralization of the charges is neglected.

As we are concerned primarily with the effects of charge, the following simplifying assumptions are made: 1) the gas is of constant composition and obeys the perfect gas law; 2) the particles are spherical, inert, and occupy a negligible volume. There is no partial pressure due to the particles, and collisional effects are neglected; 3) in order to specify the heat transfer to the particle we put the Nusselt number, equal to 2, where $N = 2H\sigma/k$. The temperature of the particle is assumed to be uniform; and 4) over-all charge neutrality exists in the asymptotic states and the charge per particle is a constant of the flow. The validity and consequences of these assumptions have been thoroughly discussed in the references, except for (4), which is clearly consistent with the most common mechanisms for producing charges on the dust.

Equations and Analysis

For the gas we have the conservation of mass

$$\rho u = \dot{m} \tag{1}$$

for the ith particle species

$$\rho_{pi} u_{pi} = K_i \dot{m} \tag{2}$$

The particle mass density may be expressed by

$$\rho_{pi} = (4\pi/3)\sigma_i^3 \rho' n_{pi} \tag{3}$$

Conservation of momentum for the gas phase may be written

$$\rho u \frac{du}{dx} + \frac{dP}{dx} = -\sum_i \tfrac{3}{8}\dot{m}K_i \frac{C_{Di}}{\sigma_i}\frac{\rho}{\rho'}\frac{|u-u_{pi}|(u-u_{pi})}{u_{pi}} \tag{4}$$

where the summation is the total aerodynamic force on the gas due to the particles.

The equation of motion for the particles becomes

$$\frac{du_{pi}}{dx} = \frac{3}{8}\frac{C_{Di}}{\sigma_i}\frac{\rho}{\rho'}\frac{|u-u_{pi}|(u-u_{pi})}{u_{pi}} + \frac{Q_i E}{M_i u_{pi}} \tag{5}$$

In accord with assumption (4), conservation of energy for the gas becomes

$$\rho u C_{pg}\frac{dT}{dx} - \frac{udP}{dx} = \sum_i \tfrac{3}{8}\dot{m}\frac{K_i C_{Di}}{\sigma_i u_{pi}}\frac{\rho}{\rho'}|u_{pi}-u|^3 + \sum_i n_{pi} k 4\pi\sigma_i (T_{pi} - T) \tag{6}$$

and for the ith particle species

$$dT_{pi}/dx = -4k\pi\sigma_i(T_{pi} - T)/M_i u_{pi} C_{pi} \tag{7}$$

Poisson's equation written in terms of E, the electric field in the x-direction is given as

$$dE/dx = \sum_i (n_{pi} Q_i/\varepsilon_o) \tag{8}$$

If Eqs. (4) and (5) are combined and continuity applied, then

$$\rho u \frac{du}{dx} + \frac{dP}{dx} + \dot{m}\sum_i K_i \frac{du_{pi}}{dx} - \sum_i n_{pi} Q_i E = 0 \tag{9}$$

Combining Eqs. (8) and (9) and integrating yields

$$\dot{m}\left(u + \sum_i K_i u_{pi}\right) + P - \frac{\varepsilon_o E^2}{2} = \text{constant} \tag{10}$$

Likewise, Eqs. (4–7) may be combined and integrated to give

$$C_{pg} T + \frac{u^2}{2} + \sum_i K_i\left[C_{pi} T_{pi} + \frac{u_{pi}^2}{2}\right] - \frac{3}{4}\left(\sum_i \frac{K_i Q_i}{\pi\sigma_i^3\rho'}\right)\int_{-\infty}^{x} E dx = \text{constant} \tag{11}$$

The last term on the left-hand side of Eq. (11) represents the work done in moving charged particles from the upstream asymptotic state ($-\infty$) to x. Applying Eqs. (10) and (11) between the upstream and downstream asymptotic states where the velocity and temperature of the particles approach those of the gas, and the electric field vanishes, yields

$$\dot{m}u_{-\infty}\left(1 + \sum_i K_i\right) + P_{-\infty} = \dot{m}u_{+\infty}\left(1 + \sum_i K_i\right) + P_{+\infty} \tag{12}$$

and

$$\frac{C_{pg} + \sum_i K_i C_{pi}}{R\left(1 + \sum_i K_i\right)}\frac{P_{-\infty}}{\rho_{-\infty}} + \frac{u_{-\infty}^2}{2} = \frac{C_{pg} + \sum_i K_i C_{pi}}{R\left(1 + \sum_i K_i\right)}\frac{P_{+\infty}}{\rho_{+\infty}} + \frac{u_{+\infty}^2}{2} + u_{+\infty}\sum n_{pi\infty} q_i \frac{\phi_{+\infty} - \phi_{-\infty}}{\left(1 + \sum_i K_i\right)} \tag{13}$$

Results and Discussion

Asymptotic charge neutrality, the case of most practical interest, requires the last terms of Eqs. (10) and (12) be equal to zero. It is seen that the asymptotic states of the gas are then related by the standard Rankine-Hugoniot relations for a gas with "effective" density, gas constant, and specific heat ratio as shown by others for the uncharged case.[1,2] Therefore, the effect of charge is observed only in the structure of the relaxation zone.

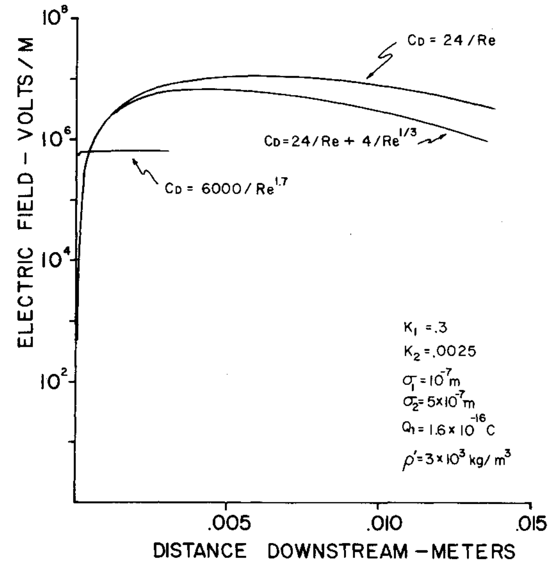

Fig. 1 Growth of the electric field for three different drag laws.

A numerical study of the governing equations was conducted in order to examine the nature of charge effects. For simplicity the investigation was restricted to two different size particles rather than a distribution. The larger was taken to be 0.5μ and the smaller 0.1μ. The large and small particles are considered to have equal and opposite charges of 1.6×10^{-16} coul.

Figure 1 shows the growth of the electric field for the following three different drag correlations: Stokes, $C_D = 24/Re$; Standard $C_D = 24/Re + 4/Re^{1/3}$; and Rudinger[12] $C_D = 6000/Re^{1.7}$.

The upstream Mach number is taken as 1.5 and the gas approximates air at room temperature and pressure. Initially, there is no charge separation so the field is zero. The small particles attempt to relax to the gas velocity in a very short time compared to the large particles. Thus, an electric field is generated due to the difference in charge density.

Figure 2 shows the particle velocities for both the charged and uncharged case for Stokes' drag law. Other conditions are the same as for Fig. 1. The particles show some oscillatory

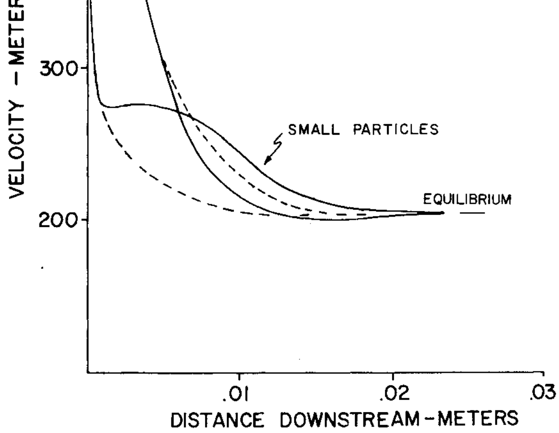

Fig. 2 Particle velocities assuming Stokes drag law. Dashed lines show velocities for the uncharged case.

behavior moving in response to the electric field. Oscillations in a system such as this are similar to those encountered in shock structure in a plasma.[13]

Generally, the effects of charge on the particles in the relaxation zone is small. For many cases of interest the effect on the gas motion is negligible as long as the loading and charge is kept low. For drag correlations other than Stokes drag law and particles larger than a micron the particle motions are only slightly affected. These facts suggest that the electric field generated by the shock might be utilized as a diagnostic technique, although it must be remembered that charge and size of the particle will be complicated distribution.

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Optimum Design of Damped Vibration Absorbers Over a Finite Frequency Range

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Introduction

A GREAT deal of attention has recently been given to application of optimization theory and techniques to a variety of engineering design problems involving dynamic response of mechanical systems. Hamad[1] considered optimum design of a dynamic absorber, using a direct search technique for the solution. Wilmert and Fox[2] have considered optimum design of a class of linear, multi-degree of freedom shock isolation systems. Schmit and Fox[3] and Schmit and Rybicki[4] treated dynamic response optimization of a simple shock isolation system, using a steepest descent method with alternate steps. In this paper, optimum design of a dynamic vibration absorber is treated by an optimal design formulation that enforces performance constraints over a range of excitation frequencies. A steepest descent programing technique is used to solve the resulting optimal design problem.[5,6]

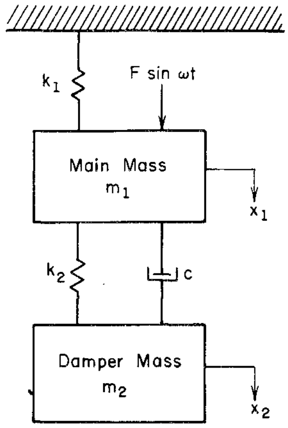

Fig. 1 Dynamic absorber.

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Optimal Design Formulation

In many practical situations, the main mass of a system may undergo large amplitude vibration, especially when the exciting frequency is close to the resonant frequency of the system. There are many techniques of reducing the amplitude of these vibrations,[7] one of which is to attach a secondary mass system to the main mass. This secondary mass system is known as the absorber system. The main mass of the system is subjected to a forcing function of frequency ω.

The dynamic absorber considered is shown in Fig. 1. The analysis of dynamic behavior of this system can be found in many textbooks.[7] The notation used in the statement and analysis of the problem is defined as follows: $x_{st} = F/k_1$ is the static deflection of the main mass, produced by force F, $\Omega_n = (k_1/m_1)^{1/2}$ is the uncoupled natural frequency of the main system, $\omega_n = (k_2/m_2)^{1/2}$ is the uncoupled natural frequency of the damper system, $\mu = m_2/m_1$ is the ratio of the masses of the absorber and the main system, $f = \omega_n/\Omega_n$ is the ratio of the uncoupled natural frequencies of the absorber and the main mass, $g = \omega/\Omega_n$ is the ratio of the exciting frequency to the uncoupled natural frequency of the main mass, $C_c = 2m_2\Omega_n$ is the critical damping, c = damping coefficient, $\xi = c/C_c$ is the damping ratio, $x_1 = x_1(\xi, f, g)$ is extreme displacement of mass 1, and $x_2 = x_2(\xi, f, g)$ is extreme displacement of mass 2. Two optimal design problems for this system can be defined as follows.

Problem 1: For a given g, find the ξ and f that minimize the ratio of extreme displacement of the main mass to its static displacement

$$J_1 = x_1(\xi, f, g)/x_{st} \tag{1}$$

subject to the "rattle space" and extreme value design variable constraints

$$\left|\frac{x_2 - x_1}{x_1}\right| \leqq Q_{\max} \tag{2}$$

$$\xi_{\min} \leqq \xi \leqq \xi_{\max} \tag{3}$$

and

$$f_{\min} \leqq f \leqq f_{\max} \tag{4}$$

This is a relatively straightforward nonlinear programing problem that may be solved by any of a number of techniques.